

Polygon Meshes and Implicit Surfaces

February 1, 2012
Jernej Barbic
University of Southern California
<http://www-bcf.usc.edu/~jbarbic/cs480-s12/>

Polygon Meshes
Implicit Surfaces
Constructive Solid Geometry
[Angel Ch. 12.1-12.3]

1

Modeling Complex Shapes



Source: Wikipedia

- An equation for a sphere is possible, but how about an equation for a telephone, or a face?
 - polygons, parametric surfaces, or implicit surfaces
- Complexity is achieved using simple pieces
- Goals
 - Model *anything* with arbitrary precision (in principle)
 - Easy to build and modify
 - Efficient computations (for rendering, collisions, etc.)
 - Easy to implement (a minor consideration...)

2

What do we need from shapes in Computer Graphics?

- Local control of shape for modeling
 - Ability to model what we need
 - Smoothness and continuity
 - Ability to evaluate derivatives
 - Ability to do collision detection
 - Ease of rendering
- No single technique solves all problems!

3

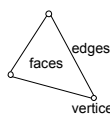
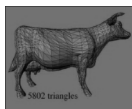
Shape Representations

Polygon Meshes
Parametric Surfaces
Implicit Surfaces

4

Polygon Meshes

- Any shape can be modeled out of polygons
 - if you use enough of them...
- Polygons with how many sides?
 - Can use triangles, quadrilaterals, pentagons, ... n-gons
 - Triangles are most common.
 - When > 3 sides are used, ambiguity about what to do when polygon nonplanar, or concave, or self-intersecting.
- Polygon meshes are built out of
 - *vertices* (points)
 - *edges* (line segments between vertices)
 - *faces* (polygons bounded by edges)



5

Polygon Models in OpenGL

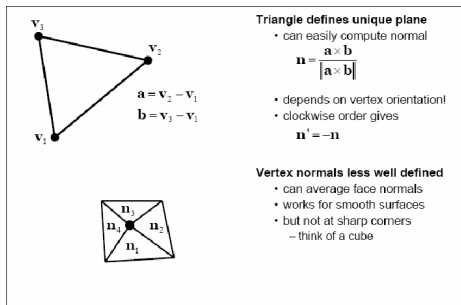
- for faceted shading

```
glNormal3fv(n);
glBegin(GL_POLYGONS);
glVertex3fv(vert1);
glVertex3fv(vert2);
glVertex3fv(vert3);
glEnd();
```
- for smooth shading

```
glBegin(GL_POLYGONS);
glNormal3fv(normal1);
glVertex3fv(vert1);
glNormal3fv(normal2);
glVertex3fv(vert2);
glNormal3fv(normal3);
glVertex3fv(vert3);
glEnd();
```

6

Normals



7

Where Meshes Come From

- Specify manually
 - Write out all polygons
 - Write some code to generate them
 - Interactive editing: move vertices in space
- Acquisition from real objects
 - Laser scanners, vision systems
 - Generate set of points on the surface
 - Need to convert to polygons



8

Data Structures for Polygon Meshes

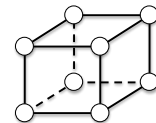
- Simplest (but dumb)
 - float triangle[n][3][3]; (each triangle stores 3 (x,y,z) points)
 - redundant: each vertex stored multiple times
- Vertex List, Face List
 - List of vertices, each vertex consists of (x,y,z) geometric (shape) info only
 - List of triangles, each a triple of vertex id's (or pointers) topological (connectivity, adjacency) info only

Fine for many purposes, but finding the faces adjacent to a vertex takes O(F) time for a model with F faces. Such queries are important for topological editing.
- Fancier schemes:
 - Store more topological info so adjacency queries can be answered in O(1) time.
 - Winged-edge data structure* - edge structures contain all topological info (pointers to adjacent vertices, edges, and faces).

9

A File Format for Polygon Models: OBJ

```
# OBJ file for a 2x2x2 cube
v -1.0 1.0 1.0      - vertex 1
v -1.0 -1.0 1.0    - vertex 2
v 1.0 -1.0 1.0     - vertex 3
...
v 1.0 1.0 1.0
v -1.0 1.0 -1.0
v -1.0 -1.0 -1.0
v 1.0 -1.0 -1.0
v 1.0 1.0 -1.0
f 1 2 3 4
f 8 7 6 5
f 4 3 7 8
f 5 1 4 8
f 5 6 2 1
f 2 6 7 3
```



Syntax:

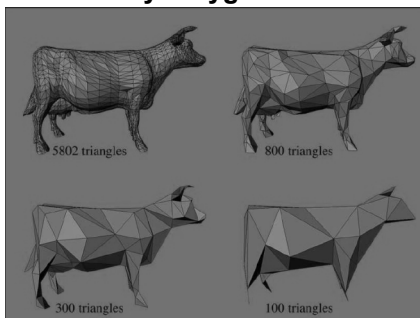
v x y z - a vertex at (x,y,z)

f $V_1 V_2 \dots V_n$ - a face with vertices $V_1, V_2, \dots V_n$

anything - comment

10

How Many Polygons to Use?



11

Why Level of Detail?

- Different models for near and far objects
- Different models for rendering and collision detection
- Compression of data recorded from the real world

We need automatic algorithms for reducing the polygon count without

- losing key features
- getting artifacts in the silhouette
- popping

12

Problems with Triangular Meshes?

- Need a lot of polygons to represent smooth shapes
- Need a lot of polygons to represent detailed shapes
- Hard to edit
- Need to move individual vertices
- Intersection test? Inside/outside test?

13

Shape Representations

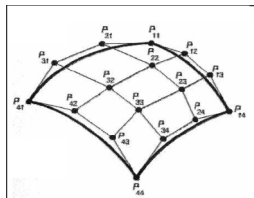
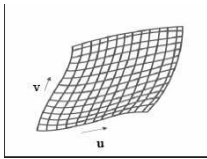
Polygon Meshes
Parametric Surfaces
Implicit Surfaces

14

Parametric Surfaces

$$p(u,v) = [x(u,v), y(u,v), z(u,v)]$$

- e.g. plane, cylinder, bicubic surface, swept surface



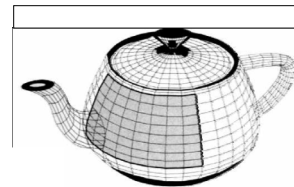
Bezier patch

15

Parametric Surfaces

$$p(u,v) = [x(u,v), y(u,v), z(u,v)]$$

- e.g. plane, cylinder, bicubic surface, swept surface



the Utah teapot

16

Parametric Surfaces

Why better than polygon meshes?

- Much more compact
- More convenient to control --- just edit control points
- Easy to construct from control points

What are the problems?

- Work well for smooth surfaces
- Must still split surfaces into discrete number of patches
- Rendering times are higher than for polygons
- Intersection test? Inside/outside test?

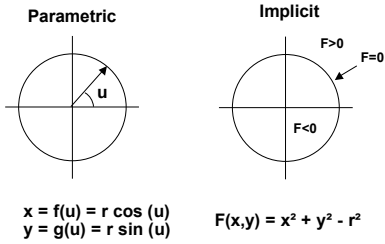
17

Shape Representations

Polygon Meshes
Parametric Surfaces
Implicit Surfaces

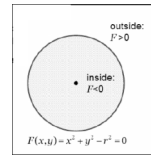
18

Two Ways to Define a Circle



19

Implicit Surfaces



- well defined inside/outside
- polygons and parametric surfaces do not have this information

• Computing is hard:
implicit functions for a cube?
telephone?

• Implicit surface: $F(x,y,z) = 0$

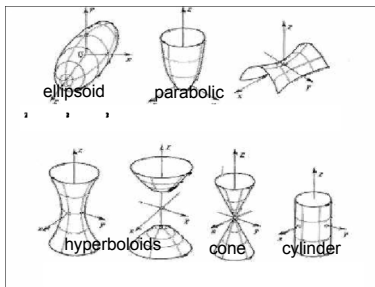
- e.g. plane, sphere, cylinder, quadric, torus, blobby models
- sphere with radius r : $F(x,y,z) = x^2 + y^2 + z^2 - r^2 = 0$

- terrible for iterating over the surface
- great for intersections, inside/outside test

20

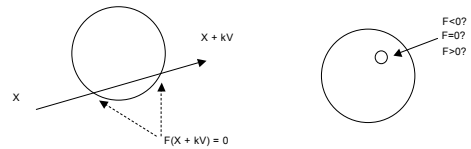
Quadric Surfaces

$$F(x,y,z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2px + 2qy + 2rz + d = 0$$



21

What Implicit Functions are Good For



Ray - Surface Intersection Test

Inside/Outside Test

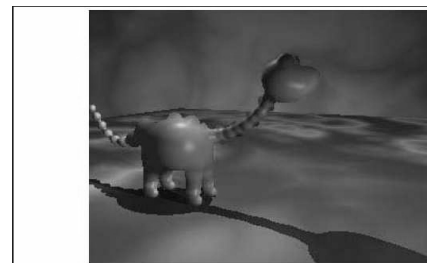
22

Surfaces from Implicit Functions

- Constant Value Surfaces are called (depending on whom you ask):
 - constant value surfaces
 - level sets
 - isosurfaces
- Nice Feature: you can add them! (and other tricks)
 - this merges the shapes
 - When you use this with spherical exponential potentials, it's called *Blobs*, *Metaballs*, or *Soft Objects*. Great for modeling animals.

23

Bloppy Models



by Brian Wyvill, <http://www.cpsc.ucalgary.ca/~blob/>

24

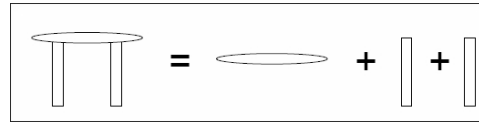
How to draw implicit surfaces?

- It's easy to ray trace implicit surfaces
 - because of that easy intersection test
- Volume Rendering can display them
- Convert to polygons: the Marching Cubes algorithm
 - Divide space into cubes
 - Evaluate implicit function at each cube vertex
 - Do root finding or linear interpolation along each edge
 - Polygonize on a cube-by-cube basis

25

Constructive Solid Geometry (CSG)

- Generate complex shapes with basic building blocks
- Machine an object - saw parts off, drill holes, glue pieces together



26

Constructive Solid Geometry (CSG)

union



the merger
of two objects
into one

difference



the subtraction
of one object
from another

intersection



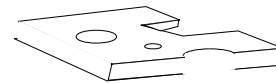
the portion
common to
both objects

Source: Wikipedia

27

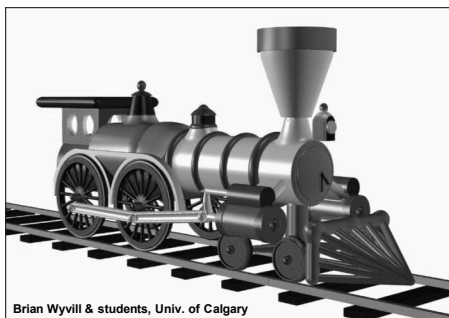
Constructive Solid Geometry (CSG)

- Generate complex shapes with basic building blocks
- Machine an object - saw parts off, drill holes, glue pieces together
- This is sensible for objects that are actually made that way (human-made, particularly machined objects)



28

A CSG Train



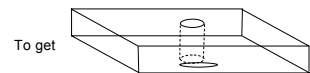
Brian Wyvill & students, Univ. of Calgary

29

Negative Objects

Use point-by-point boolean functions

- remove a volume by using a negative object
- e.g. drill a hole by subtracting a cylinder



$\text{Inside}(\text{BLOCK}-\text{CYL}) = \text{Inside}(\text{BLOCK}) \text{ And Not}(\text{Inside}(\text{CYL}))$

30

Set Operations

- UNION: $\text{Inside}(A) \parallel \text{Inside}(B)$
 ➤ Join A and B
- INTERSECTION: $\text{Inside}(A) \ \&\& \ \text{Inside}(B)$
 ➤ Chop off any part of A that sticks out of B
- SUBTRACTION: $\text{Inside}(A) \ \&\& \ (\neg \text{Inside}(B))$
 ➤ Use B to Cut A

Examples:

- Use cylinders to drill holes
- Use rectangular blocks to cut slots
- Use half-spaces to cut planar faces
- Use surfaces swept from curves as jigsaws, etc.

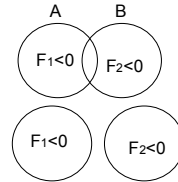
31

Implicit Functions for Booleans

- Recall the implicit function for a solid: $F(x,y,z) < 0$
- Boolean operations are replaced by arithmetic:
 - MAX replaces AND (intersection)
 - MIN replaces OR (union)
 - MINUS replaces NOT (unary subtraction)

• Thus

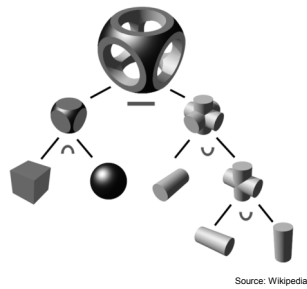
- $F(\text{Intersect}(A,B)) = \text{MAX}(F(A), F(B))$
- $F(\text{Union}(A,B)) = \text{MIN}(F(A), F(B))$
- $F(\text{Subtract}(A,B)) = \text{MAX}(F(A), -F(B))$



32

CSG Trees

- Set operations yield tree-based representation



Source: Wikipedia

33

Implicit Surfaces

- Good for smoothly blending multiple components
- Clearly defined solid along with its boundary
- Intersection test and Inside/outside test are easy
- Need to polygonize to render --- expensive
- Interactive control is not easy
- Fitting to real world data is not easy
- Always smooth

34

Summary

- Polygonal Meshes
- Parametric Surfaces
- Implicit Surfaces
- Constructive Solid Geometry

35