

CSCI 480 Computer Graphics  
Lecture 26

# Visualization

Height Fields and Contours  
Scalar Fields  
Volume Rendering  
Vector Fields

[Angel Ch. 2.11]

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<http://www-bcf.usc.edu/~jbarbic/cs480-s13/>

# Scientific Visualization

- Generally do not start with a 3D triangle model
- Must deal with very large data sets
  - MRI, e.g.  $512 \times 512 \times 200 = 50\text{MB}$  points
  - Visible Human  $512 \times 512 \times 1734 = 433\text{ MB}$  points
- Visualize both real-world and simulation data
- User interaction
- Automatic search for relevant data

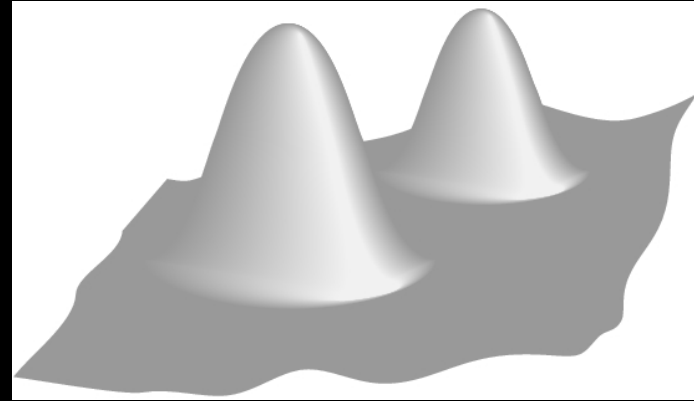
# Types of Data

- Scalar fields (3D volume of scalars)
  - E.g., x-ray densities (MRI, CT scan)
- Vector fields (3D volume of vectors)
  - E.g., velocities in a wind tunnel
- Tensor fields (3D volume of tensors [matrices])
  - E.g., stresses in a mechanical part
- Static or dynamic through time

# Height Field

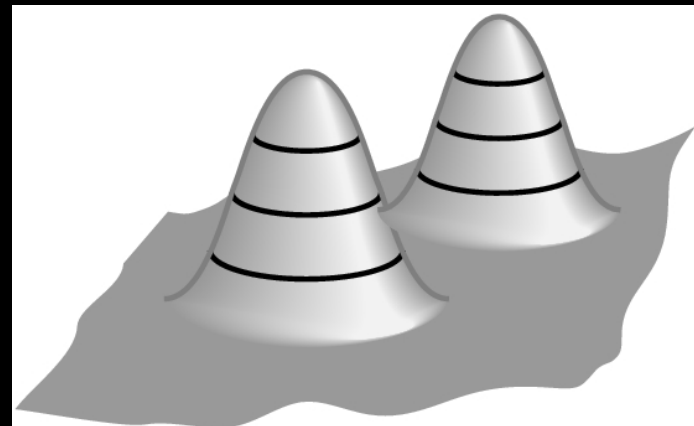
- Visualizing an explicit function

$$z = f(x,y)$$



- Adding contour curves

$$g(x,y) = c$$



# Meshes

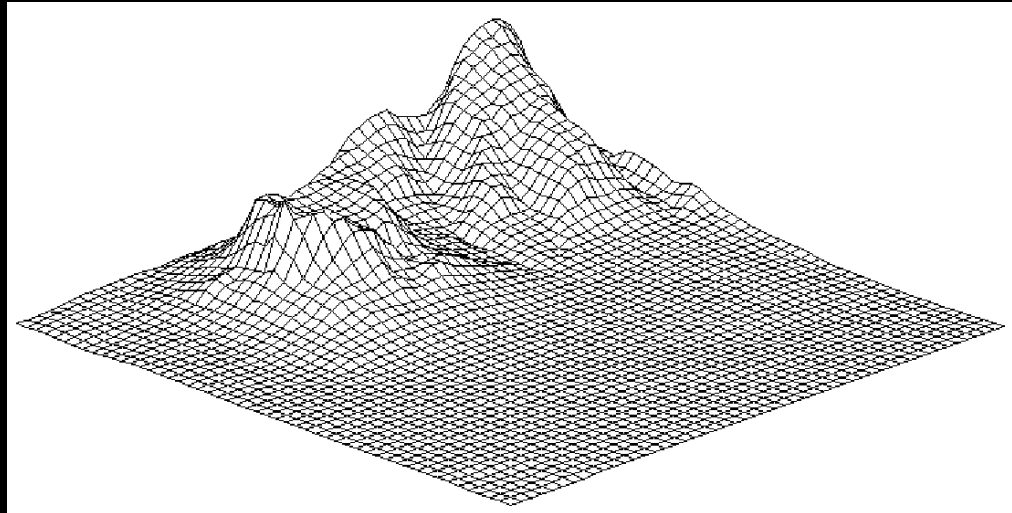
- Function is sampled (given) at  $x_i, y_j, 0 \leq i, j \leq n$
- Assume equally spaced

$$x_i = x_0 + i\Delta x$$

$$y_j = y_0 + j\Delta y$$

$$z_{ij} = f(x_i, y_j)$$

- Generate quadrilateral or triangular mesh
- **[Assignment 1]**



# Contour Curves

- Recall: implicit curve  $f(x,y) = 0$
- $f(x,y) < 0$  inside,  $f(x,y) > 0$  outside
- Here: contour curve at  $f(x,y) = c$
- Implicit function  $f$  sampled at regular intervals for  $x,y$

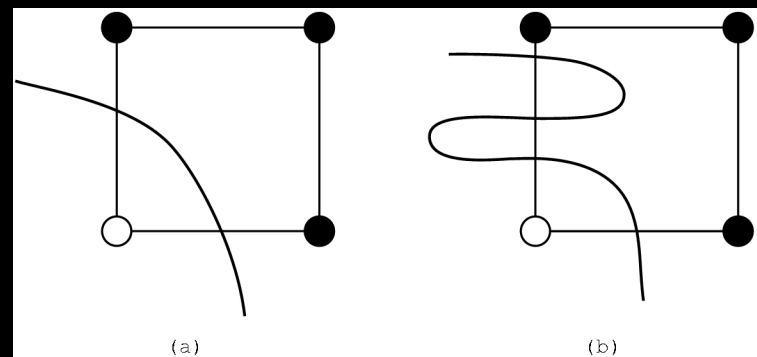
$$x_i = x_0 + i\Delta x$$

$$y_j = y_0 + j\Delta y$$

- How can we draw the curve?

# Marching Squares

- Sample function  $f$  at every grid point  $x_i, y_j$
- For every point  $f_{ij} = f(x_i, y_j)$  either  $f_{ij} \leq c$  or  $f_{ij} > c$
- Distinguish those cases for each corner  $x$ 
  - White:  $f_{ij} \leq c$
  - Black:  $f_{ij} > c$
- Now consider cases for curve
- Assume “smooth”

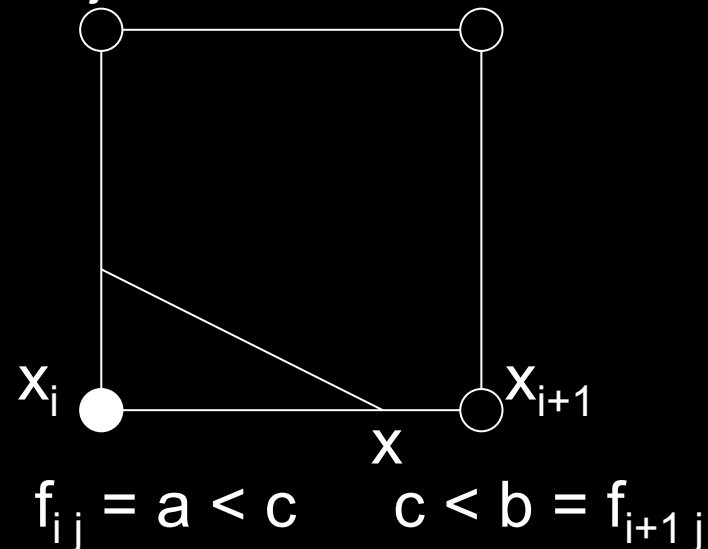


# Interpolating Intersections

- Approximate intersection
  - Midpoint between  $x_i, x_{i+1}$  and  $y_j, y_{j+1}$
  - Better: interpolate
- If  $f_{ij} = a$  is closer to  $c$  than  $b = f_{i+1j}$  then intersection is closer to  $(x_i, y_j)$ :

$$\frac{x - x_i}{x_{i+1} - x} = \frac{c - a}{b - c}$$

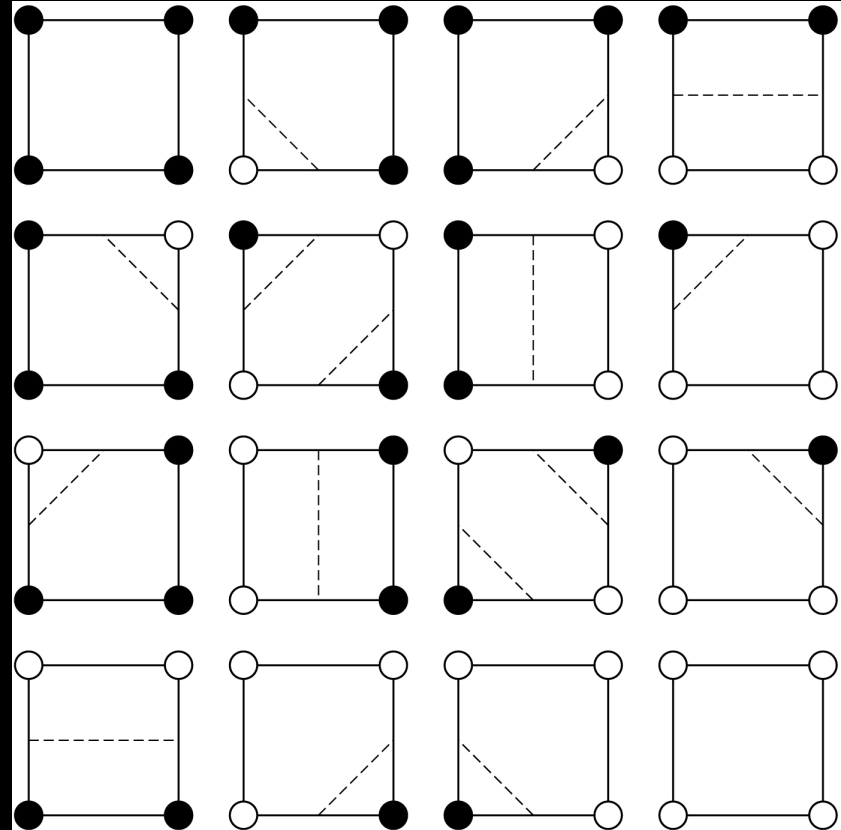
- Analogous calculation for  $y$  direction



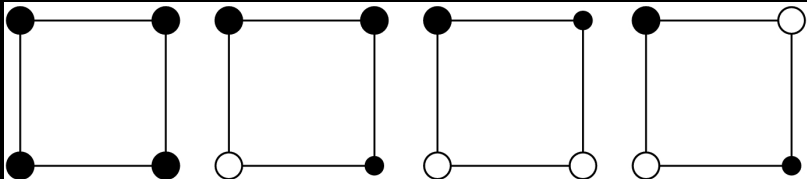


# Cases for Vertex Labels

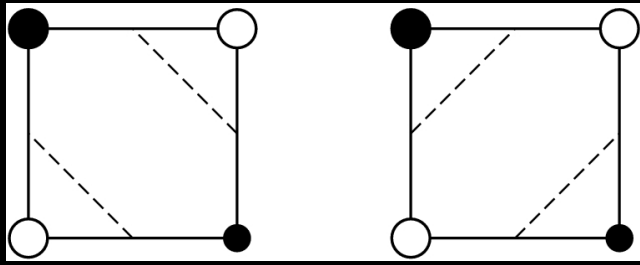
16 cases for vertex labels



4 unique cases  
modulo symmetries

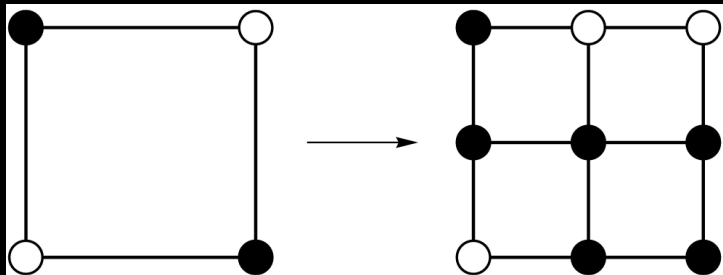
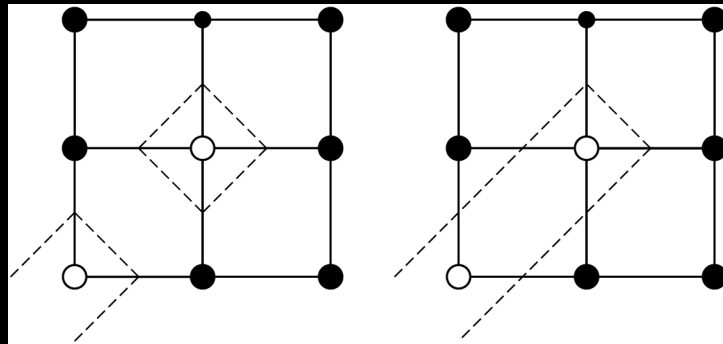


# Ambiguities of Labelings



Ambiguous labels

Different resulting contours

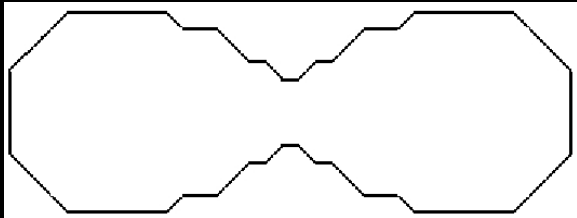


Resolution by subdivision  
(if such higher resolution  
available / possible)

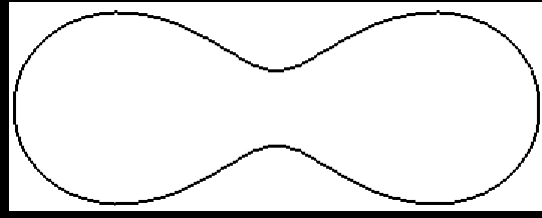
# Marching Squares Examples

- Ovals of Cassini, 50 x 50 grid

$$f(x, y) = (x^2 + y^2 + a^2)^2 - 4a^2x^2 - b^4$$
$$a = 0.49, b = 0.5$$

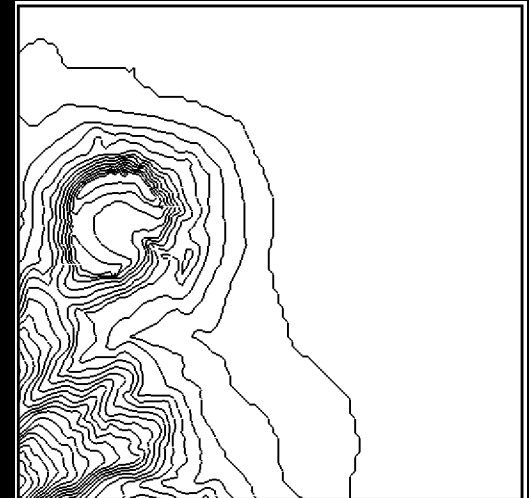


Midpoint



Interpolation

Contour plot of Honolulu data



# Outline

- Height Fields and Contours
- **Scalar Fields**
- Volume Rendering
- Vector Fields

# Scalar Fields

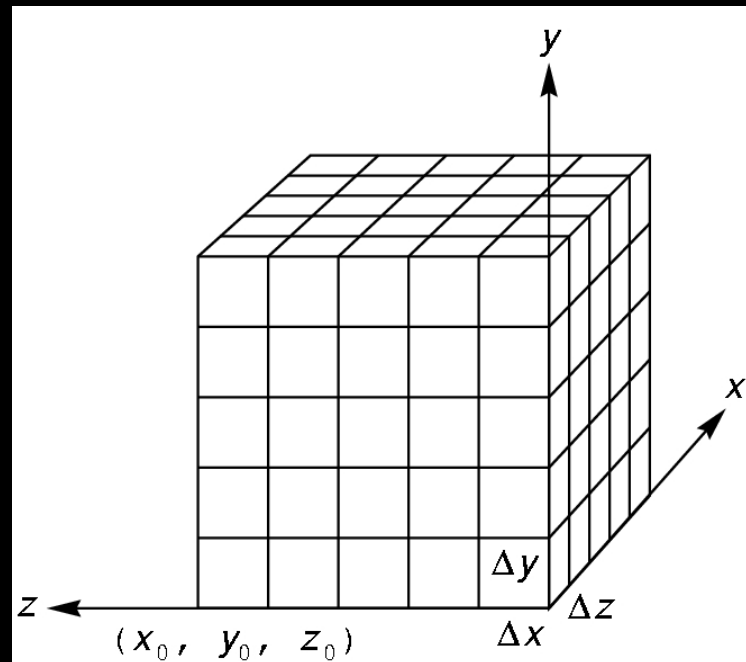
- Volumetric data sets
- Example: tissue density
- Assume again regularly sampled

$$x_i = x_0 + i\Delta x$$

$$y_j = y_0 + j\Delta y$$

$$z_k = z_0 + k\Delta z$$

- Represent as **voxels**

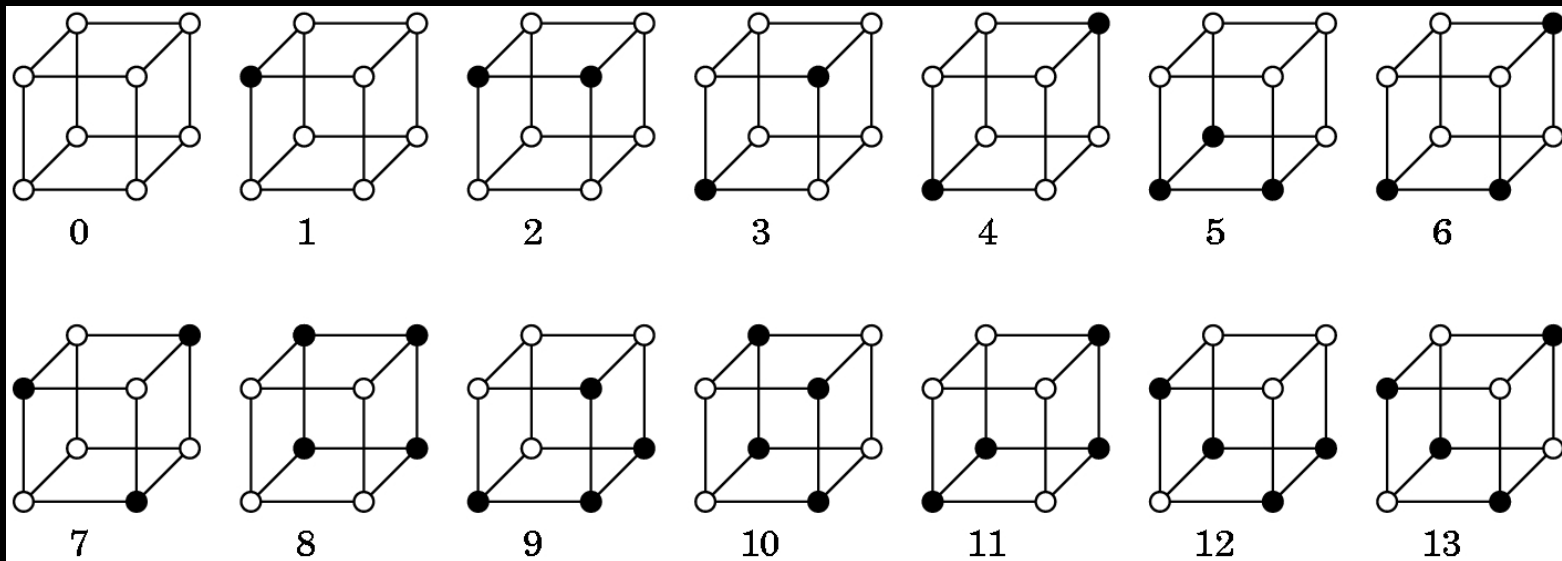


# Isosurfaces

- $f(x,y,z)$  represents volumetric data set
- Two rendering methods
  - Isosurface rendering
  - Direct volume rendering (use all values [next])
- **Isosurface** given by  $f(x,y,z) = c$
- Recall implicit surface  $g(x, y, z)$ :
  - $g(x, y, z) < 0$  inside
  - $g(x, y, z) = 0$  surface
  - $g(x, y, z) > 0$  outside
- Generalize right-hand side from 0 to  $c$

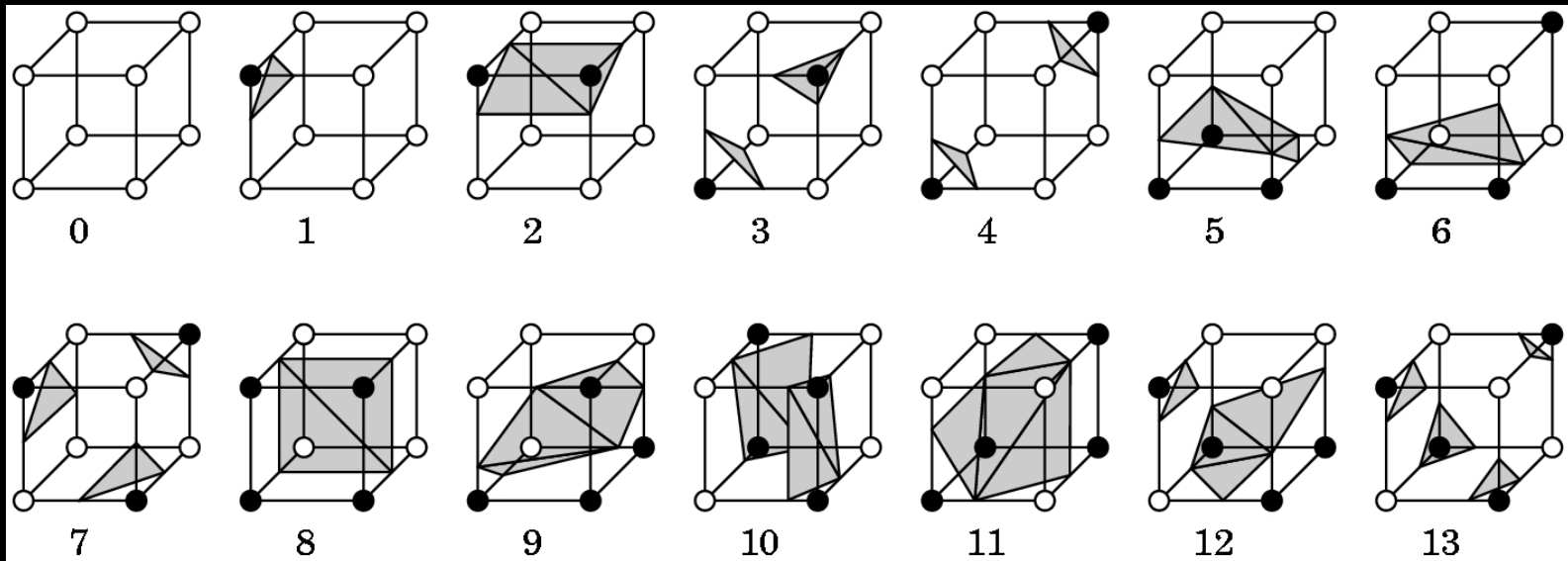
# Marching Cubes

- Display technique for isosurfaces
- 3D version of marching squares
- 14 cube labelings (after elimination of symmetries)



# Marching Cube Tessellations

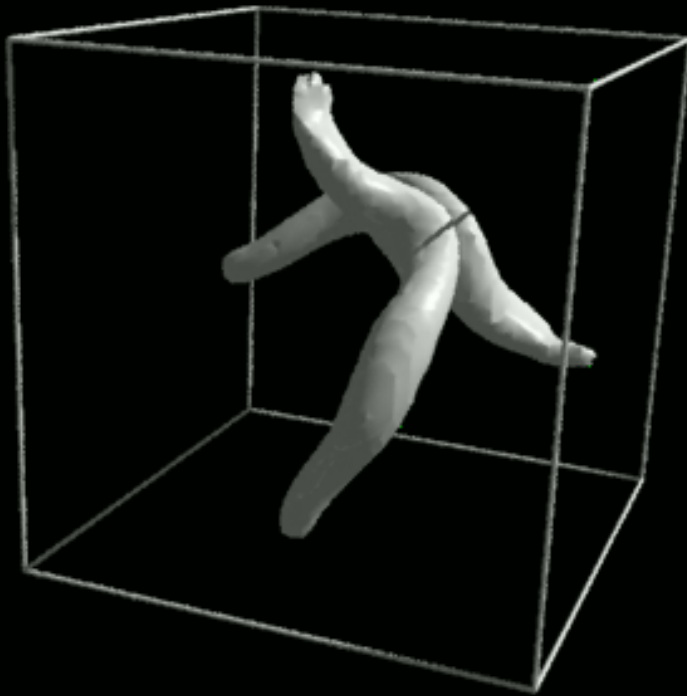
- Generalize marching squares, just more cases
- Interpolate as in 2D
- Ambiguities similar to 2D



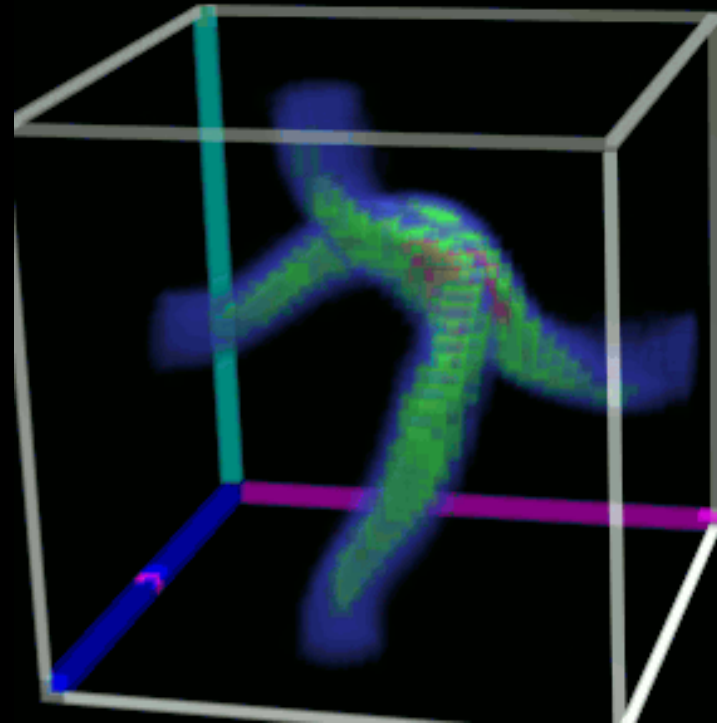


# Volume Rendering

- Sometimes isosurfaces are unnatural or do not give sufficient information
- Use all voxels and transparency ( $\alpha$ -values)



Ray-traced isosurface



Volume rendering

# Surface vs. Volume Rendering

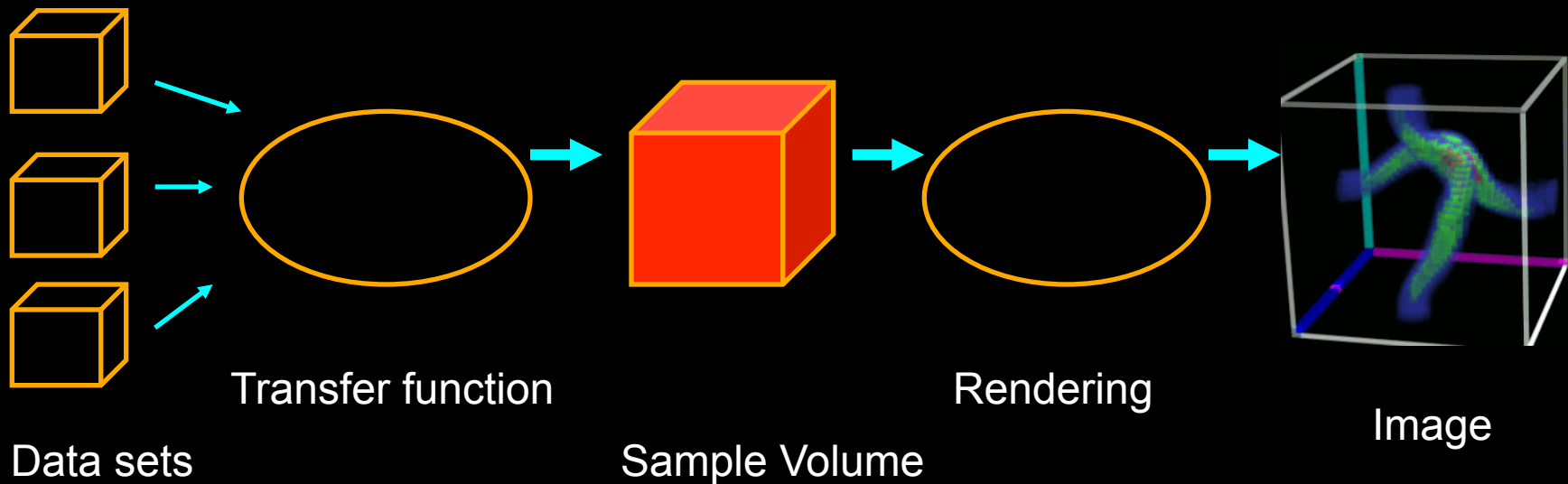
- 3D model of surfaces
- Convert to triangles
- Draw primitives
- Lose or disguise data
- Good for opaque objects
- Scalar field in 3D
- Convert it to RGBA values
- Render volume “directly”
- See data as given
- Good for complex objects

# Sample Applications

- Medical
  - Computed Tomography (CT)
  - Magnetic Resonance Imaging (MRI)
  - Ultrasound
- Engineering and Science
  - Computational Fluid Dynamic (CFD)
  - Aerodynamic simulations
  - Meteorology
  - Astrophysics

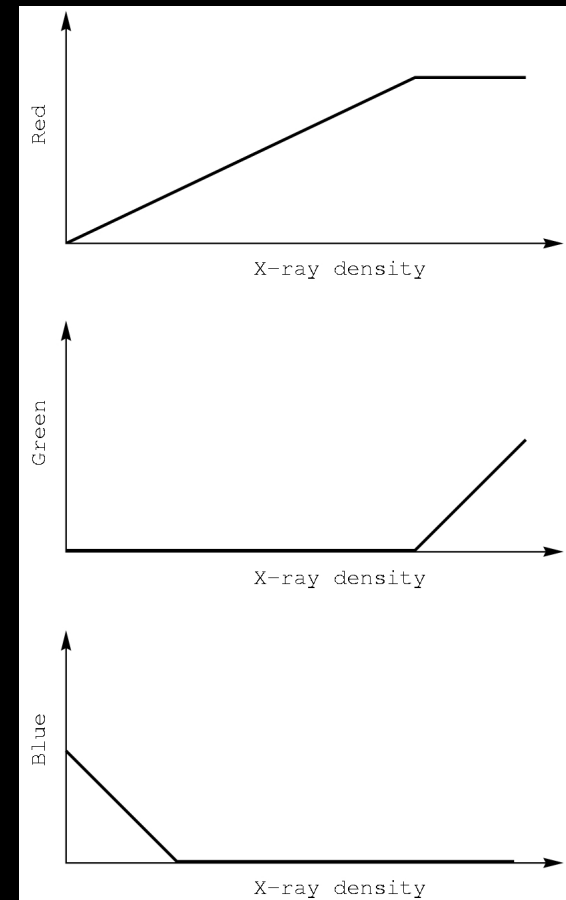
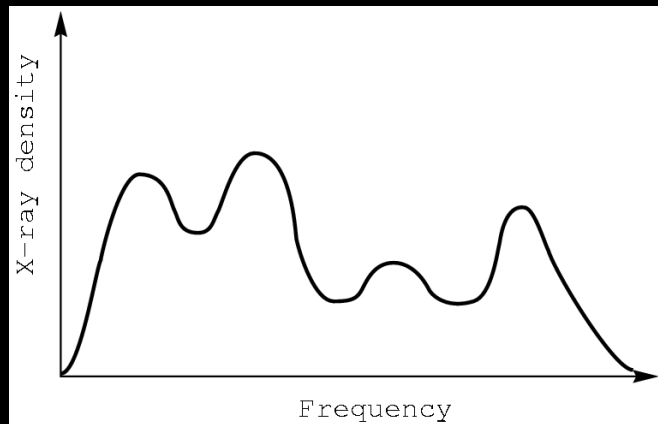
# Volume Rendering Pipeline

- Transfer function: converts input data set to colors and opacities
  - Example input:  $256 \times 256 \times 256 \times 8$  bytes = 128 MB
  - Convert to 24 bit color, 8 bit opacity



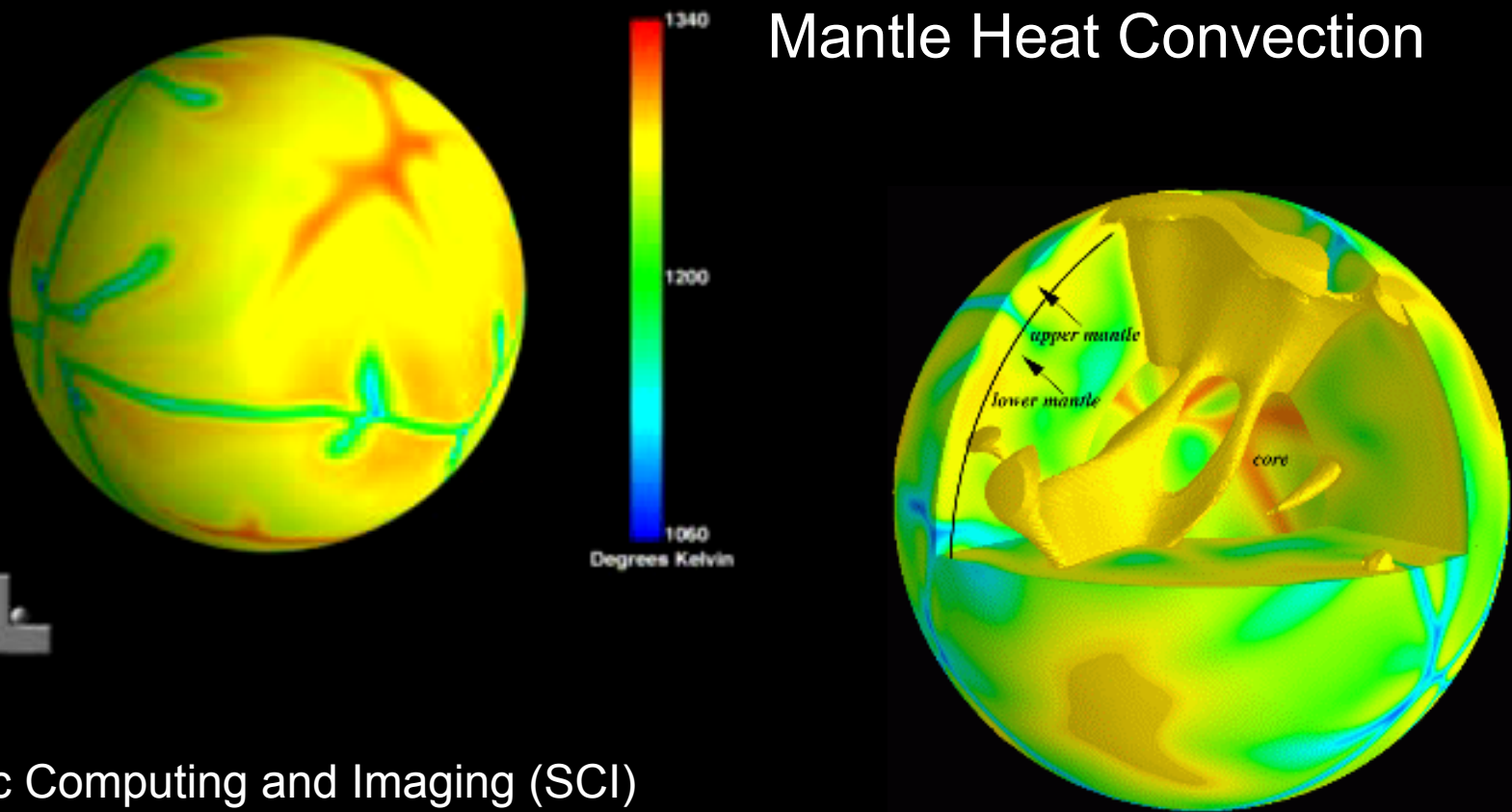
# Transfer Functions

- Transform scalar data values to RGBA values
- Apply to every voxel in volume
- Highly application dependent
- Start from data **histogram**
- Opacity for emphasis



# Transfer Function Example

## Mantle Heat Convection



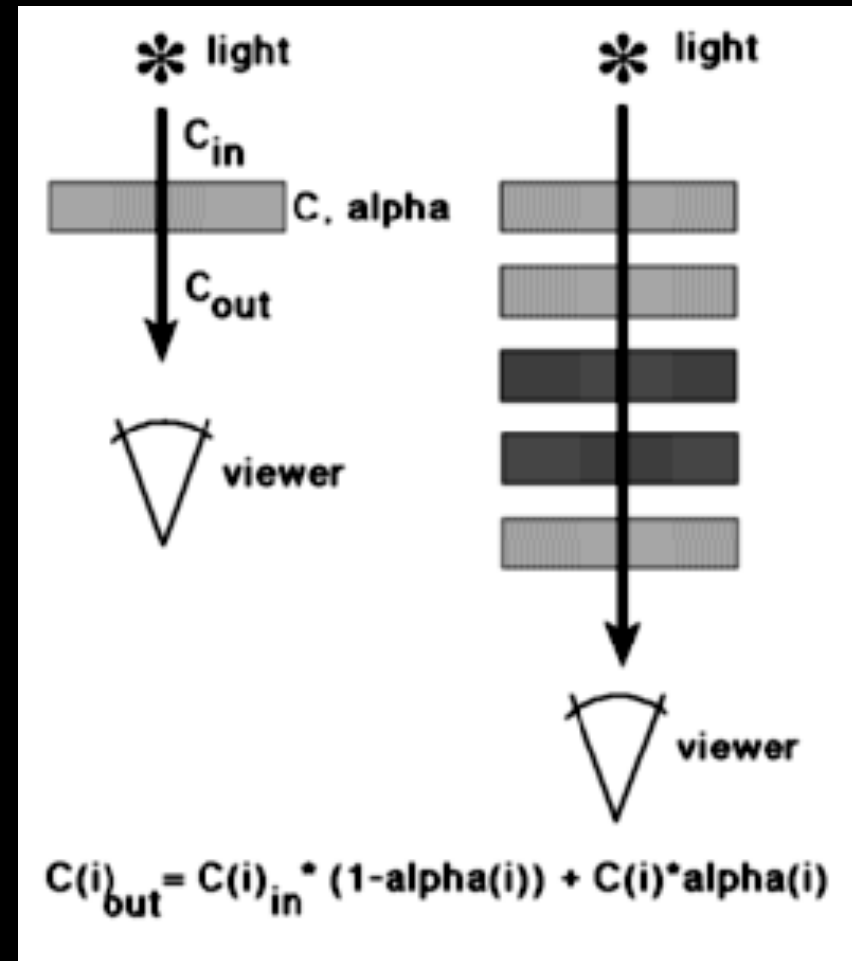
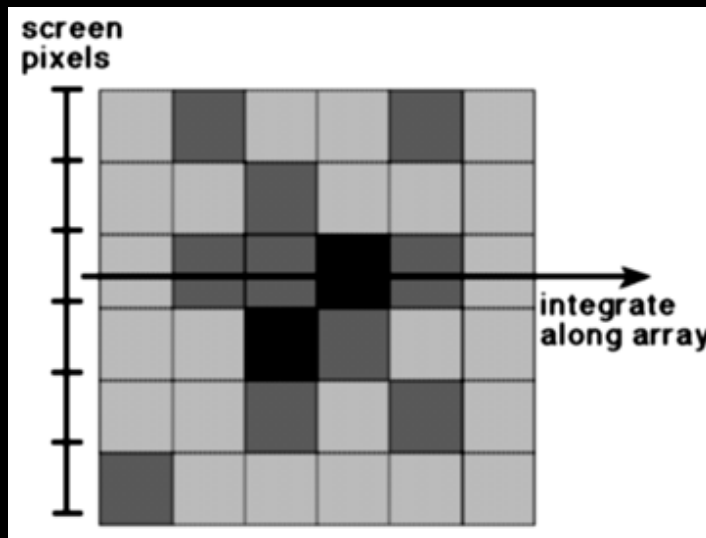
Scientific Computing and Imaging (SCI)  
University of Utah

# Volume Ray Casting

- Three volume rendering techniques
  - Volume ray casting
  - Splatting
  - 3D texture mapping
- Ray Casting
  - Integrate color through volume
  - Consider lighting (surfaces?)
  - Use regular x,y,z data grid when possible
  - Finite elements when necessary (e.g., ultrasound)
  - 3D-rasterize geometrical primitives

# Accumulating Opacity

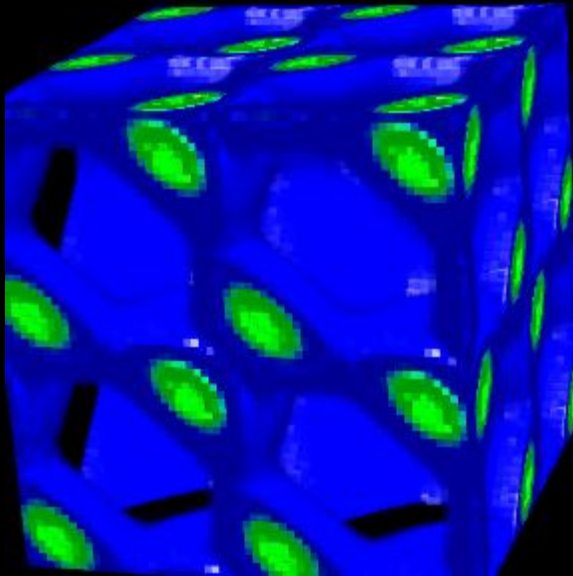
- $\alpha = 1.0$  is opaque
- Composite multiple layers according to opacity
- Use local gradient of opacity for enhanced display of boundaries



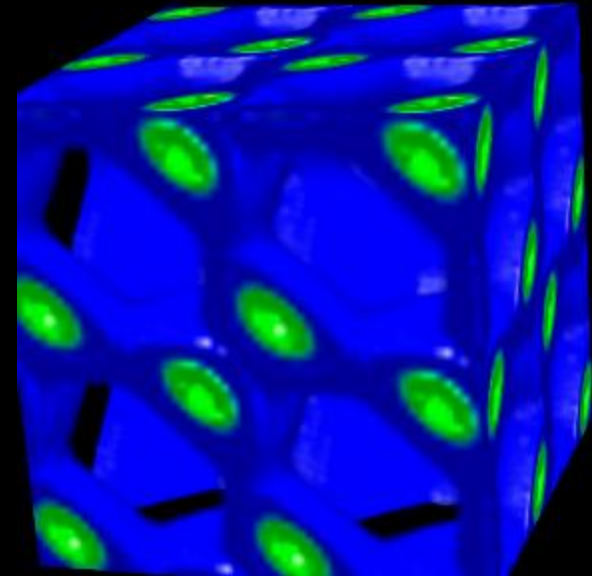


# Trilinear Interpolation

- Interpolate to compute RGBA away from grid
- Nearest neighbor yields blocky images
- Use **trilinear interpolation**
- 3D generalization of bilinear interpolation



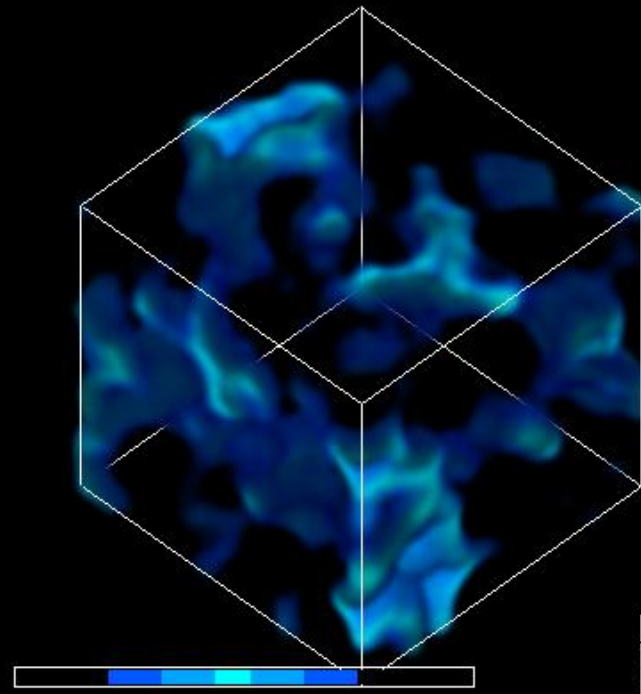
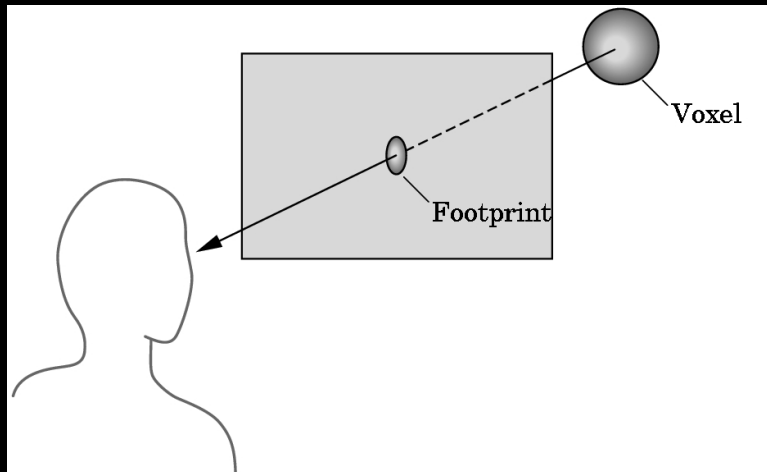
Nearest  
neighbor



Trilinear  
interpolation

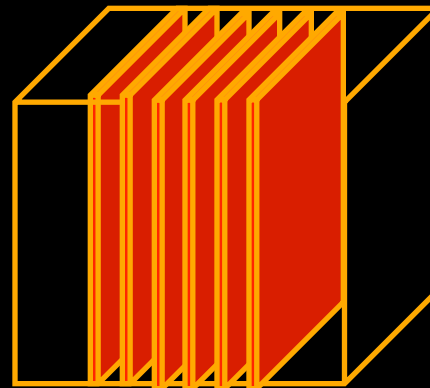
# Splatting

- Alternative to ray tracing
- Assign shape to each voxel (e.g., Gaussian)
- Project onto image plane (**splat**)
- Draw voxels back-to-front
- Composite ( $\alpha$ -blend)



# 3D Textures

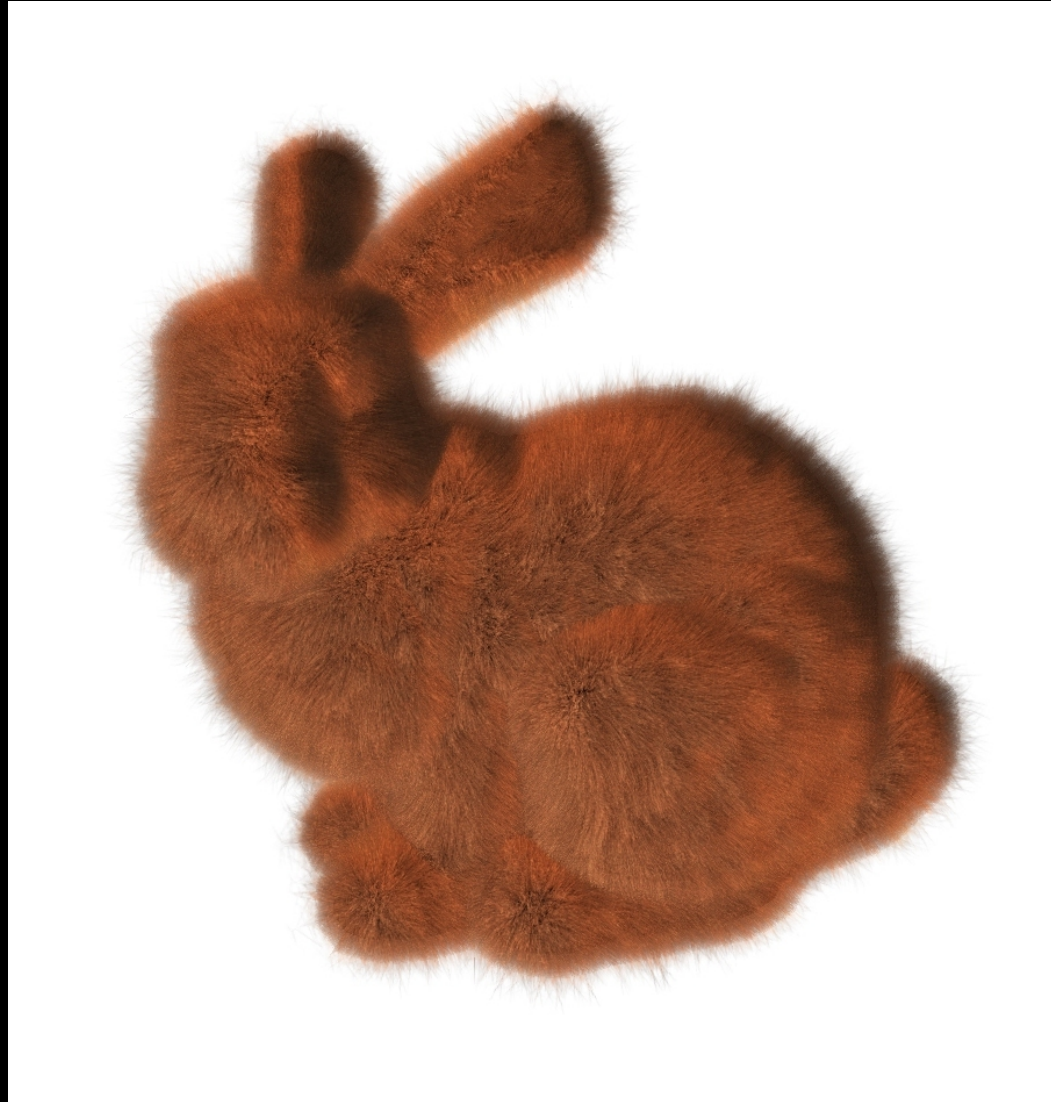
- Alternative to ray tracing, splatting
- Build a 3D texture (including opacity)
- Draw a stack of polygons, back-to-front
- Efficient if supported in graphics hardware
- Few polygons, much texture memory



Draw back to front

3D RGBA texture

# Example: 3D Textures



Emil Praun' 01

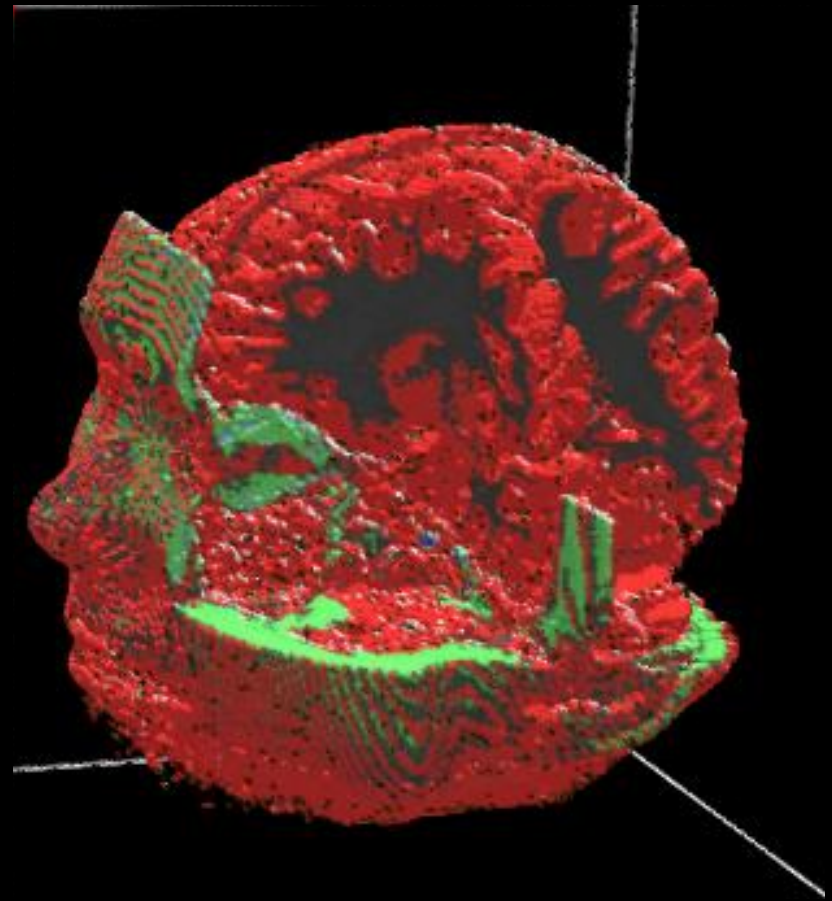
# Example: 3D Textures

Emil Praun' 01



# Other Techniques

- Use CSG for cut-aways



# Acceleration of Volume Rendering

- Basic problem: Huge data sets
- Must program for locality (cache)
- Divide into multiple blocks if necessary
  - Example: marching cubes
- Use error measures to stop iteration
- Exploit parallelism

# Outline

- Height Fields and Contours
- Scalar Fields
- Volume Rendering
- **Vector Fields**



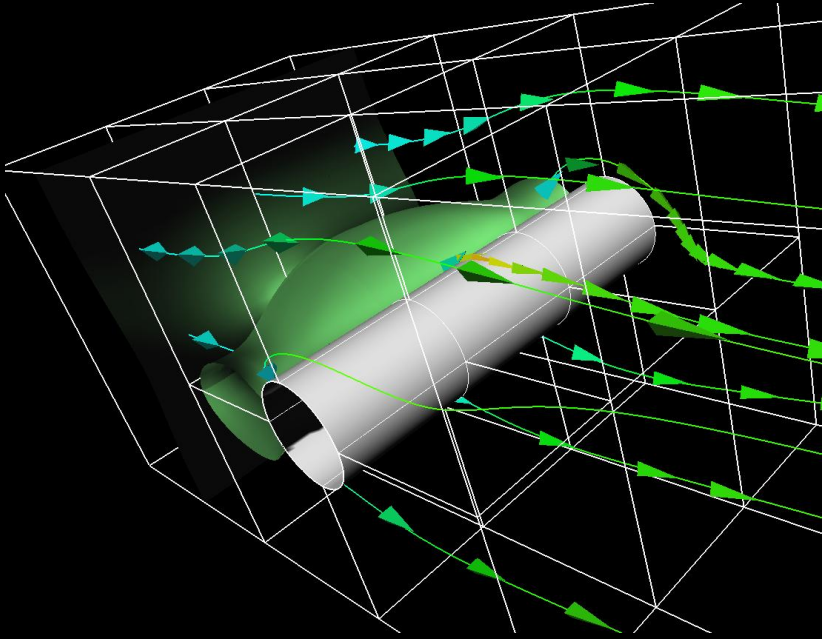
# Vector Fields

- Visualize vector at each  $(x,y,z)$  point
  - Example: velocity field
  - Example: hair
- **Hedgehogs**
  - Use 3D directed line segments (sample field)
  - Orientation and magnitude determined by vector
- Animation
  - Use for still image
  - Particle systems

Blood flow in  
human carotid artery

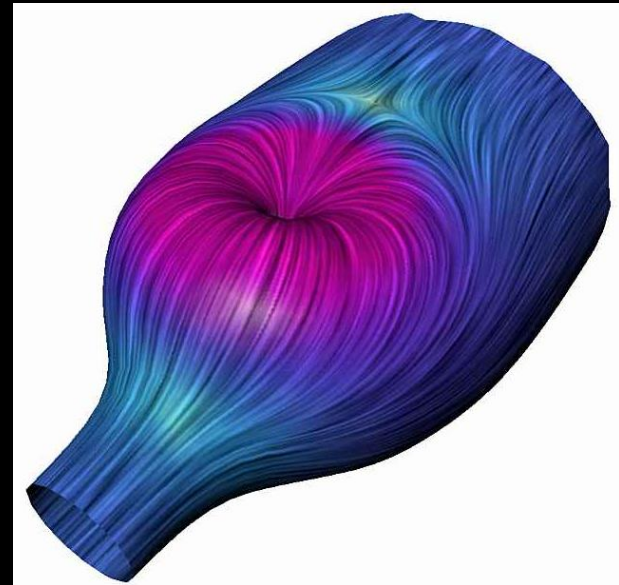
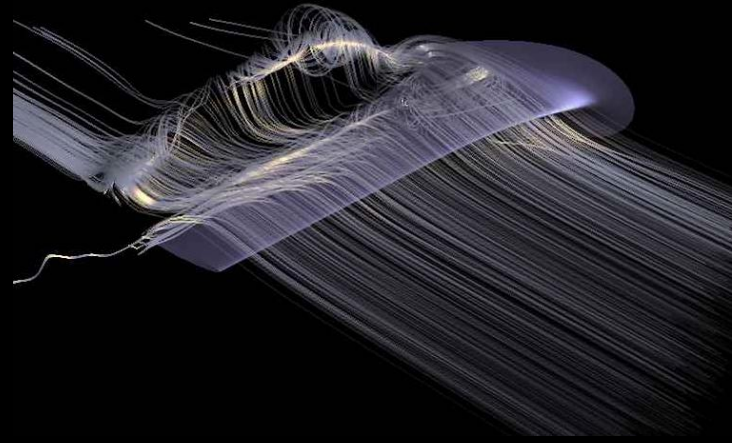


# Using Glyphs and Streaklines

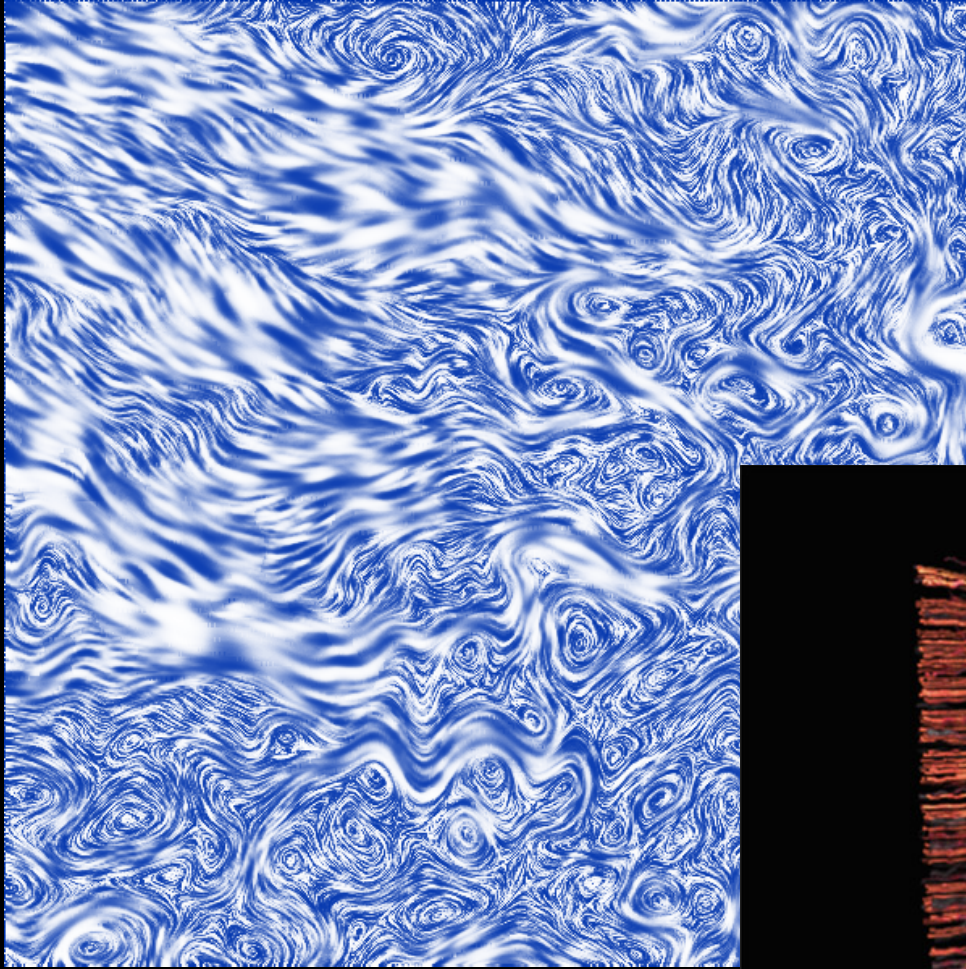


Glyphs for air flow  
University of Utah

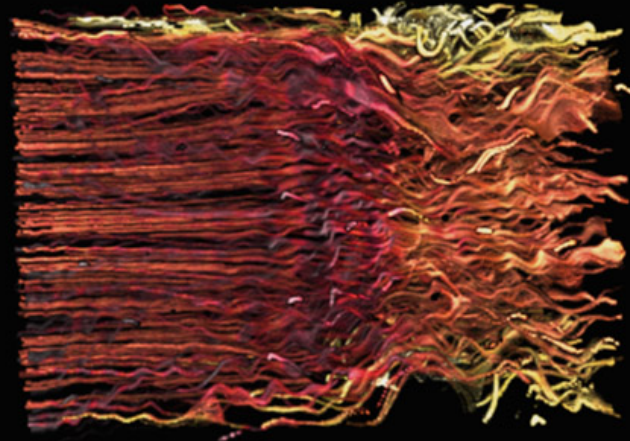
Glyph = marker (for example,  
an arrow) used for data visualization



# More Flow Examples

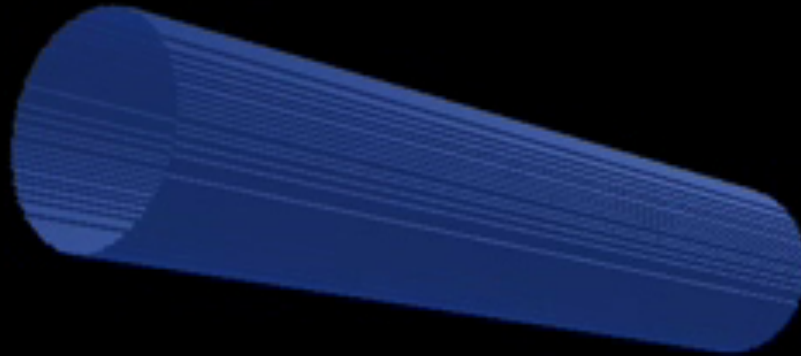


Banks and Interrante



# Example: Jet Shockwave

P. Sutton  
University of Utah



<http://www.sci.utah.edu/>

# Summary

- Height Fields and Contours
- Scalar Fields
  - Isosurfaces
  - Marching cubes
- Volume Rendering
  - Volume ray tracing
  - Splatting
  - 3D Textures
- Vector Fields
  - Hedgehogs
  - Animated and interactive visualization